

*Full Length Research Paper*

## Constructive metacognitive activity shift in mathematical problem solving

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Received 02 March, 2016; Accepted 7 April, 2016

This study aims to describe the constructive metacognitive activity shift OF eleventh graders in solving a mathematical problem. Subjects in this study were 10 students IN grade 11 of SMAN 1 Malang. They were divided into 4 groups. Three types of metacognitive activity undertaken by students when completing mathematical problem are awareness, regulation, and evaluation. This was a descriptive qualitative research with exploratory approach. Data collection instruments used were mathematical problem solving task of Model Eliciting Activities (MEA) and interview. In the first stage, students completed MEA mathematical problems individually while doing think aloud. In the second stage, students worked in small groups consisting of three students to solve the same problem and then the researchers recorded the results of the discussion by using a handycam. In the third stage, the researchers conducted task-based interviews to determine the metacognitive activity performed by students when solving problems either completed them individually or in group. Based on the analysis results of students' work, think aloud, students' conversations record group during discussion and interviews, the data show that three students spread into 4 groups of constructive metacognitive activity shift.

**Key words:** Metacognitive activity, socially metacognition, constructive metacognitive activity shift, problem solving.

### INTRODUCTION

Metacognition and problem solving are intimately connected. Metacognition has important role in mathematical problem solving (Aurah et al., 2011; Biryukov, 2014; Kapa, 2002; Wilson and Clarke, 2002; Wismath et al., 2014). Metacognition in problem solving helps problem solver identify the problems to be solved, helps look back what exactly the problems are, and help you better understand how to achieve a goal or solution

(Kuzle, 2013).

The definition of metacognition has improved a lot in the education literature. In general, metacognition is defined as thinking about what people think (Dawson, 2008; Schoenfeld, 1987; Shetty, 2014), but this definition does not sufficiently explore students' metacognition when solving mathematical problems. Some experts more specifically define that metacognition is one's

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knowledge about cognitive processes and one's awareness on a mathematical problem involving the process of planning, monitoring, and evaluation of problem solutions (Flavell, 1976; King et al., 1993; Magiera and Zawojewski, 2011; Wilson and Clarke, 2002). Based on some of these opinions, it can be concluded that metacognition is thinking about what has been thought associated with a person's awareness and the ability to evaluate and organize their own thinking.

Mental activity associated with metacognition is referred to as metacognitive activity. Mental activity is a process occurring in the mind which can be seen through the behavior of students in solving problems. Such behavior can be a statement expressed by students when solving problems. The statement expressed by students in solving problems can be a result of thinking aloud and group discussion.

Metacognitive activity is also growing in the social environment. Magiera and Zawojewski (2011) state that individual metacognitive activity can be developed through social interaction in which the discussion or conversation can serve as a tool supporting the emergence of metacognitive activity (Magiera and Zawojewski, 2011). Metacognitive activity in social interaction requires a reciprocal relationship and involvement of other members in a group to solve the problems together. Some researchers have revealed that metacognitive activity influenced by social interaction (Chiu and Kuo, 2010; Goos et al., 2002; Hurme et al., 2009; Magiera and Zawojewski, 2011).

The studies that have been conducted by experts still evidently have not revealed about the shifting process of metacognitive activity from individual to social in solving mathematical problems. Influences in group discussions result in metacognitive activity shifting process from individual to social, that is, a condition in which students to re-examine their mathematical thinking and revise their initial solutions in solving the problems. Therefore, the previous studies still have not revealed the shifting process of metacognitive activity from individual to social. The objective of this study is to describe the shifting process of metacognitive activity from individual to social in solving mathematical problems.

Metacognitive activity shifts can be grouped into two, namely constructive and perfective metacognitive activity shifts. Constructive metacognitive activity shift involves the metacognitive activity components of awareness and evaluation. Perfective metacognitive activity shift involves the metacognitive activity components of awareness, evaluation, and regulation.

### **Study objective**

The objective of this study was to describe the constructive metacognitive activity shift of students in solving mathematical problems. In this context, the study

aims to answer the following questions:

1. How are students' thinking structure in solving problems individually based on three components of metacognitive activity, namely awareness, regulation, and evaluation?
2. How are students' thinking structure in solving problems in group based on three components of metacognitive activity, namely awareness, regulation, and evaluation?
3. How is the scheme of constructive metacognitive activity shift of students in solving problems from individual to social?

## **REVIEW OF LITERATURE**

### **Metacognitive activities in problem solving**

Three elements of metacognitive activity in problem solving by Wilson and Clarke (2002, 2004) are awareness, regulation, and evaluation. Metacognitive awareness is related to one's awareness where he or she is in the process of problem solving, problem-solving strategies that can be brought to respond the problems, and the relationship between prior knowledge and specialized knowledge required to solve the problems. Metacognitive evaluation is a decision made by the problem solvers related to their own thinking, limitations of one's thinking about the problem situation, limitations of one's strategy to solve the problems, for instance one can make assessment about the effectiveness of their thinking or choosing their strategies. Metacognitive regulation refers to a person's knowledge concerning the selection and use of strategies that include how and why they use certain strategies and use skills such as planning, self-correcting, and setting goals. Indicators of each metacognitive activity awareness, regulation, and evaluation presented by Wilson and Clarke (2002) can be seen in Table 1.

To explore the metacognitive activity in problem solving, it also needs to pay attention on the types of problems that will be used.

According to Baker and Cerro (2000), problems chosen for the metacognitive activity study include difficult, complex, new tasks, and require metacognitive skills to complete. We use Model Eliciting Activities (MEA) because its characteristics are consistent with the recommendations of Baker and Cerro. MEA is a kind of open problems requiring the development of mathematical models and requiring enough challenge that members of the group should be involved to decide, test, and revise the initial solution that in turn will lead to the monitoring, effectiveness evaluation of the initial solution, and making decisions (Magiera and Zawojewski, 2011).

### **Metacognitive activity shift**

Metacognitive activity shift involves activity shift from individual to social. Metacognitive activity shift occurs

**Table 1.** Indicators of each metacognitive activity by Wilson and Clarke (2002).

Metacognitive Activity	Indicator
<b>Awareness</b>	1. I thought about what I already know
	2. I tried to remember if I had ever done a problem
	3. like this before
	4. I thought about something I had done another time
	5. that had been helpful
	6. I thought 'I know what to do'
	7. I thought 'I know this sort of problem'
<b>Regulation</b>	1. I made a plan to work it out
	2. I thought about a different way to solve the problem
	3. I thought about what I would do next
	4. I changed the way I was working
<b>Evaluation</b>	1. I thought about how I was going
	2. I thought about whether what I was doing was working
	3. I checked my work
	4. I thought 'Is this right?'
	5. I thought 'I can't do it'

when students are getting the influence of group discussion so that they check their mathematical thinking and revise their initial solutions in resolving problems. This is in line with what is expressed by Magiera and Zawojewski (2011) that individual metacognitive activity can be developed through social interaction in which languages can serve as a tool that supports the internalization of metacognitive activity that is initially targeted at others.

Previous research has studied a lot of problem solver metacognitive activity in solving mathematical problems related to social interaction (Goos, 2002; Goos et al., 2002; Iiskala et al., 2004; Magiera and Zawojewski, 2011), but these studies still do not reveal the process of metacognitive activity shift from individual to social in solving mathematical problems. Magiera and Zawojewski (2011) only identify and characterize the social-based and self-based context associated with metacognitive activity encoded as awareness, regulation, and evaluation. Magiera and Zawojewski (2011) state that transcript segment is coded as social-based when students show that they re-examine their own mathematical thinking as a result of direct interaction with members of the group. Then, explanation in segment transcript is identified as self-based when subject metacognitive activity is generated internally rather than occurring as a result of interaction with members of the group.

In exploring the metacognitive activity shift of individual to social, the researchers need to consider two important things, namely student think aloud and conversation when discussing to solve problems. Mokos and Kafoussi (2013) state that the verbal reports obtained from the think aloud show the metacognitive strategies used by

students when they solve problems. The conversation among the group members during the discussion to solve problems provides an opportunity to better explore the thoughts of students, and is a place that is rich in metacognitive activities (Hurme et al., 2009; Magiera and Zawojewski, 2011). Therefore, to explore the metacognitive activity in this study, we use the results of think aloud and conversations recording of students when having group discussion. In the future, if there are things that need to be explored further, the researchers should conduct unstructured interviews with the subject.

## METHODOLOGY

### Subject

The participants were 12 students of grade 11 of SMAN 1 Malang divided into 4 groups. Each group consisted of 3 students with heterogeneous skills in solving problems together, in which previously every student solves problems individually. Having obtained saturation data in the subject, they were 10 subjects who experienced metacognitive activity shift and 2 students who did not experience metacognitive activity shift.

### Instruments

There were two main types of instruments and auxiliary instrument used in this work. The main instruments were the researchers themselves who act as planners, data collectors, data analysts, data interpreters, and reporters of the research results. The auxiliary instruments used in this study were mathematical problem solving tasks, interview guides, individual and social questionnaires.

### Problem solving task

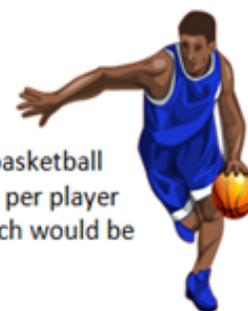
Mathematical problem-solving task used in this study was MEA

### Problem

The next year's National Basketball Tournament will be held in Gelanggang Istora Senayan/ Istora Senayan Arena Jakarta on 15-16 June 2016. If you as the manager of basketball team is interested in attending this national basketball tournament, calculate the cost per player and the total registration fee and hotel. Please rank the hotel provided and decide which would be best for your team by considering the entire data on the table.

Pay attention to the following data:

Your team has 9 players interested in attending the tournament and 1 coach for a total of 10 people. The registration fee is Rp. 1.500.000,00 per team.



No	Hotel	The average price per night per room	Maximum people per room	The distance of tournament venue from hotel	The number of restaurant near the hotel	Hotel facilities
1	Millennium Hotel*	IDR 1.090,000	4 people	10 km	4	Pool only
2	Pradana Hotel	IDR 630.000	2 people	15 km	7	Pool and play ground
3	Santika Premiere Hotel*	IDR 1.320,000	4 people	5 km	2	Play ground only

\* Free transport fee to the match location at Gedung Olahraga Istora

Figure 1. Mathematical problem solving task.

(Model eliciting Activities) type. MEA intended in this study was an open problem related to the decision to choose which one was the best. MEA problems related to decision-making would encourage students to test, set a strategy, revise their initial solution, and evaluate the effectiveness of initial solutions they had made, and make a final decision. At first, the students were given the opportunity to work on this MEA problem individually for 20-30 min and then later the students worked in small groups consisting of 3 people to solve the same problem. Here is MEA problem related to the decision making given on the students. Mathematical problem-solving task is presented in Figure 1.

### Questionnaire

The purpose of this questionnaire was to indicate whether or not the students' metacognitive activity appear when solving problems and then it was confirmed through interviews. The questionnaire was adapted from Biryukov (2014), which was subsequently adapted to the objective of our study. The questionnaire in this study consisted of 11 statements directed to three categories: awareness, regulation, and evaluation. Students got the questionnaire after they had completed the task of solving math problems individually and as a group after finishing the problems. Students were asked to provide the check mark "√" in the column corresponding to their activity when solving problems.

### Interview

Interview guide was used as a reference when the researchers

conducted interviews with the subject. The purpose of the interview was to confirm the results of students' think aloud, conversation during the discussion, and questionnaire. The interview used was unstructured interview, meaning that the questions were prepared in advance but in the field the questions were adjusted to the conditions. Interviews were conducted after students solve the problems in groups.

### Procedure

In the first stage, students completed mathematical problem solving tasks individually while doing think aloud. In the second stage, students discussed and made decisions together in small groups consisting of three students to solve the same problem and then the researchers recorded the results of the discussion by using a handy cam. In the third phase, the researchers conducted task-based interviews and confirmed metacognitive activity appearing when solving problems either individually or in groups

### Data analysis

This study is a qualitative research with descriptive exploratory approach. At the data analysis stage, the activities conducted by researchers were (1) transcribing the data obtained from think aloud and student conversations record when having group discussion and interview), (2) data reduction including explaining, choosing principal matters, focusing on important things, removing the unnecessary ones, and organizing raw data obtained from the field, (3) Encoding the data, including to take written data that had

been collected, segmenting the sentences into categories, then labeling these categories with specific terms. Encoding data from think aloud, student conversations record group during discussion, and interviews refers to indicators of each metacognitive activity awareness, regulation, and evaluation presented by Wilson and Clarke (2002), (4) describing the constructive metacognitive activity shift in mathematical problem solving, and (5) conclusion.

## RESULTS

From the 10 subjects obviously there are 3 students who experience constructive metacognitive activity shift and 7 students who experience perfective metacognitive activity shift with the same characteristics. Next, will be described one subject, namely S3 who experience constructive metacognitive activity shift, because this shift involves the three components of metacognitive activity namely awareness, evaluation, and regulation as well as constructive metacognitive activity shift makes student change the complex answer.

### ***The thinking structure of S3 when solving problems individually***

At this stage of understanding the problem, S3 did not immediately understand what the problem is. Students need to read many times to be able to understand what is being asked back in a basketball tournament problem because the question in this basketball tournament problem does not appear explicitly. This fact can be seen from the results S3 think aloud when solving the problems and interview transcript conducted by researchers with the S3. Here are the results of S3 think aloud.

**S3:** *(start to read the problem) ... What does it mean? (Reread the problem). Emmm ... it means there is the cost per player, the total cost of the tournament and the hotel, making the hotel ranking is provided. So, the first determines the fee per player, then 1,500,000 for registration fee per team (awareness).*

Furthermore, the results of interviews with S3 are as follows.

**I:** *After reading the last issue, do you immediately understand the meaning of this problem?*

**S3:** *First I read all, then I leave it whether I understand or not, after that I read the problem again. So, I need to read many times, Mam.*

**I:** *What makes you read this problem many times?*

**S3:** *Finding the questions Mam.*

**I:** *Furthermore, when you are reading this problem repeatedly what do you rethink?*

**S3:** *So, the point is we are told to find the cost per player.*

### ***the total cost of the tournament, and the hotel as well as the hotel ranking. (Awareness)***

Based on the results of think aloud and interview, it appears that after repeatedly reading and rethinking the problem, the new S3 realizes that the problem is to find the cost per player, the total cost of the tournament and the hotel as well as the hotel ranking. S3 also doing a regulation which is rethinking the strategy in determining the cost of the tournament and the hotel, cost per player, and hotel ranking. The strategy done by S3 is to decide the hotel directly; the hotel that became his first choice is the one with the most inexpensive price per night. Here are the results of S3 think aloud while considering and rethinking about the hotel ranking.

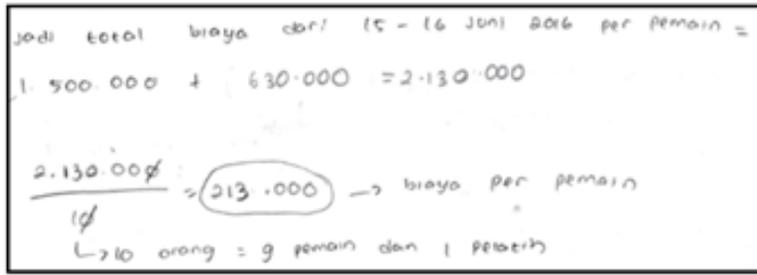
**S3:** *Which becomes the first choice is the Santika premiere hotel because it was the closest. But ... (followed by whispering, thinking hard to associate with other criteria such as price, maximum person per room, the number of restaurants, and facilities). No...no...I should not choose Santika premiere first. So the first option is Pradana hotel because it is the cheapest among 3 other hotels, the maximum number of people per room is only a few which means players can rest a little bit more, there are many restaurants, a pool and playground that can be used for training (evaluation).*

Based on the results of think aloud, S3 is still not sure (still in a state of disequilibrium) on his first choice hotel, that is Santika premiere. This is because, S3 is still questioning and re-considering other criteria such as price, maximum person per room, the number of restaurants, and facilities. Moreover, after completing an evaluation by rethink about his decision in deciding the best hotel ranking. S3 prioritize on the cheapest hotel price, thus Pradana hotel became the first choice hotel.

Based on the results of think aloud above, it appears that S3 also conducts regulations which determine the hotel which becomes his first choice based on the distance. At the beginning S3 thought that his first choice hotel is the Santika premiere, but is still unsure about the choice; therefore he conducted an evaluation by reconsidering about his decision in determining the best hotel ranking. The reason of S3 to decide which hotel to be the first choice is if the best hotel has been known, he will determine the hotel price and regulation fee as well as the cost per player easily. This fact is also strengthened from the interview result between the researchers and S3.

**I:** *How do you solve it?*

**S3:** *First, find his hotel, so I choose Pradana hotel because the price is cheaper than the other, per room only consists of 2 people, so if per room only allowed for 2 people, the players can take a rest better, then there*



Handwritten calculation:

$$1.500.000 + 630.000 = 2.130.000$$

$$\frac{2.130.000}{10} = 213.000 \rightarrow \text{biaya per pemain}$$

↳ 10 orang = 9 pemain dan 1 pelatih

Printed calculation:

So the total cost of 15-16 June 2016 per player  
 $1.500.000 + 630.000 = 2.130.000$   
 $\frac{2130000}{10} = 213.000$  the cost per player

Figure 2. The work of S3.

are 7 restaurants, swimming pool and playground as a place to exercise.

*I: Why do you choose this strategy?*

*S3: That is because for example if we determine the hotel first I became confused, since if you try to find the cost per player you must find the total cost first. We would not be able to find the total cost if we did not know the hotel price per night; we also would not know the total price of the hotel. Then, if we have known the hotel price per night we could add the regulation fee, after that the tournament cost is found, divided by many players and from here will find the fee per player (regulation).*

The next step performed by S3 is implementing a strategy of determining the total cost of the tournament for June 15-16 can be seen in Figure 2. The total cost of the tournament is obtained from the registration fee plus the lodging cost for 1 night so that  $1,500,000 + 630,000 = 2.13$  million. Furthermore, to determine the cost per player is  $2,130,000 / 10 = 213,000$  (divided by 10 because there are 9 players and 1 coach). To rank the hotel, S3 determines that the first choice hotel is pradana, the second choice is the Santika Premiere Hotel arguing that even though they were very expensive but the distance from the hotel to the tournament venue is close, and the third is the Millennium Hotel because of the distance from the tournament venue plus the price is quite expensive, and the facilities are only few which is swimming pool.

During problem resolving, S3 encountered an error and attempted to correct it back. This fact is obtained from the questionnaire and think aloud which is further strengthened from the interview result. Based on the results of the questionnaire, S3 described his fault that when calculating the cost of the hotel, he forgot to multiply the cost of the hotel with how long the team to stay and have not multiplied yet with many needed rooms. S3 realizes that he made a mistake that can also be seen from his think aloud result.

**S3:** For hotel ranking the first choice is Pradana hotel, the second is Santika premiere hotel because even though it is expensive the distance from the hotel to the

tournament venue is close, the third is millennium hotel because the price is quite expensive, the distance from the tournament venue is far, and a few facility ...emmm.. ... wait a minute (pause while thinking back about hotel ranking) ... yeah, it is still unfinished .... my previous calculations on the cost of the hotel was wrong (awareness). Earlier, it was minus, it should be  $1.500.000 + (10/2 \times 630,000) = 4.65$  million due to pradana hotel with maximum 2 people per room there so we need 5 rooms. Then, for a fee per player are  $4.65$  million / 10 = 465,000.

From the think aloud results it appears that S3 determines the hotel ranking; he conducted the evaluation by think again about the cost of hotel rooms and registration fees. Next, when S3 evaluating the cost of a hotel room, S3 is aware that the procedure for calculating the cost of the hotel is still not correct because it has not multiplied by how long it takes a team to stay and have multiplied again with many rooms required by the team. S3 then re-regulates the procedure in determining the cost of the tournament and the hotel and the cost per player as being strengthened by the interview result with S3.

*I: do you find an error and correct it during the problem solving?*

*S3: Yes ma'am, once I was inaccurate in counting so it makes me have to work twice. It turns out before that I haven't multiply the length of stay, this was only the main cost and I haven't multiplied by many needed rooms for 10 people yet.*

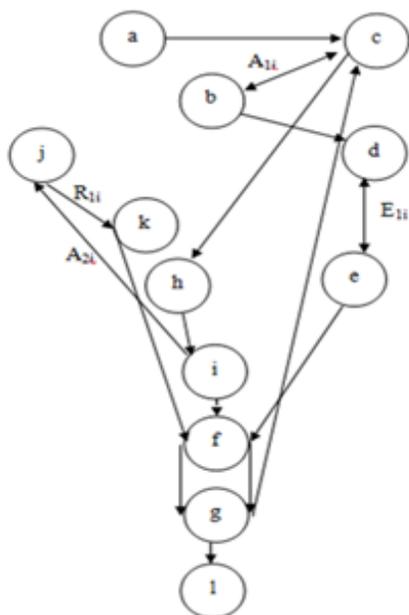
*I: Then, what are you thinking until you could find error?*

*S3: When I wrote the hotel rankings. I thought back about my hotel ranking I made which was related to the lodging cost, I just remembered oh yes there were 10 people that I haven't multiplied with many needed rooms. So I need to fix it again (awareness)*

*I: And furthermore what do you do?*

*S3: I fix it again on the procedures for fee per player and tournament fees and hotel (regulation).*

Based on think aloud and interview results, it seems that S3 realizes that he made a mistake and tried to correct it



**Figure 3.** The thinking structure of S3 when working problems individually.

back. When S3 makes a rank order of the hotel, he reevaluates that hotel ranking he made relates to the lodging cost or hotel room. Later he realized that the procedure for calculating the lodging cost is still not correct because it has not multiplied by the length of stay and the team has not been multiplied again with many needed rooms for 10 people. Further, S3 fix the procedures again for determining the cost per player and tournament fees and hotel. Furthermore, the S3 thinking structure in resolving individual problems can be seen in Figure 3.

### **The thinking structure of S3 after solving the problems in group**

Basketball tournament problem is designed to be done first individually and then after that, it is done in groups of 3 people and the time to finish is about 30 min. Basketball tournament problem is designed to require a considerable challenge that the members of the group should be involved in decision making, examining and revising their initial solutions, and encouraging the involvement of metacognitive activities of awareness, regulation, and evaluation. Discussion in small groups will have an impact on other members to examine and revise their initial solutions. Influence in this group discussion results in metacognitive activity shift process from individual to social so that students dismantle again or change the their initial procedure path in overall, that is

originally wrong, or eliminate some of the structural components of their initial thinking in solving the problems (Table 2).

At the time of group discussion, S3 tries to rethink its decision to rank the hotel and the cost per player. Here is the transcript of discussion between S3 and his 2 discussions friends, namely S1 and S2.

S2: Okay, which first hotel do you choose, friends?

S1: I choose millennium hotel, the first

S2: Same, I also choose millennium hotel, for you (S3), which one do you choose?

S3: I choose Pradana hotel

S1: Why do you choose Pradana hotel?

S3: So even though for hotel Pradana it is not free for the transportation, but the cost per night is cheaper than most of other hotels. For the match, the players need good place to rest, so I choose a maximum of two people per room so the rest can be maximum. For the transportation problems, we can bring our own car from the school. Furthermore, it has the most restaurant, and there is a pool and a playground too.

S1: How about you (S2)?

S2: my first choice is millennium hotel because based in terms of the cost per person is pretty cheap anyway, but if compared with pradana, pradana is more cheaper, but in pradana the transportation cost does not free. Just imagine that the fuel is so expensive now, especially for commuting and farthest distance is 15 km. Meanwhile in the millennium the transport is free, a maximum people of a room is 4, I think there is no problem with that than 2 people per room that we are order it will be too many rooms we were order and it will increase the costs. In the other hand what we must to do with the playground, it may not be able to drill the basketball because of fear to damage the park. Furthermore, my second choice is Santika premiere because the important thing in my opinion its free transportation cost. The Santika premiere as the most important according to me that there is free transportation to the GOR, it if more than 5 km free transportation is passable save.

S1: Yeah anyways difference in the cost of Pradana with the premiere Santika know is quite a lot above 690,000, nearly 700,000, so the millennium was mending rather than Santika because the difference is that too much can make the other more important needs.

S2: Yes I think also so, I also think to prefer the swimming pool than playground

S1: Yes the pool is more beneficial for the players to refreshing and soaking

S2: At the millennium hotel is also free transport

S1: Yeah I was thinking also is in the millennium it's the transportation cost it's free and the location is quite close to the hotel. While the distance of the Pradana it is very far away, we also anticipated if the players wake up late and the distance of Pradana hotel to the area is very far

**Table 2.** Code description of the thinking structure of S3 when working problems individually.

a : Reading the problem	j : Determining the need of room for 10 people on of each type of hotel
b : Analyzing the criteria (cost, distance, the number of restaurant, and facilities on each hotel	k : Calculating the room cost for 10 people on each type of hotel
c : Determining 1) order/hotel ranked from best to worst, 2) total cost of the tournament and the hotel, and 3) the cost per player	l : finished
d : The cheapest hotel price per night	→ : The order of activity when solve the problems
e : Determine the best hotel; that is Pradana hotel	↔ : Activities of alternating or 2 directions
f : Calculating the total cost of a hotel room and registration	A <sub>i</sub> : Individual awareness
g : Calculating the cost per player for 1 night	R <sub>i</sub> : Individual regulation
h : Considering the presence / absence of transportation costs and distance	E <sub>i</sub> : Individual evaluation
i : The order of hotel's rank from best to worst	

away and it can make big problems.

S3: yeah that its true, (awareness) if the charge transportation is not free, it will be adding the costs, considering the current fuel is expensive now, so if the transportation costs is not free it will adding the costs (Evaluation).

Based on the discussions between S3 and his two friends, it seems that S3 realizes that from his friend idea, it needs to reconsider the relationship between the hotel ranking and all the criteria provided in the table as the price per night, maximum persons per room, distance, restaurants, facilities, presence or absence of transportation costs. Furthermore, S3 do evaluation by rethinking his decision in determining the ranking of the hotel. S3 reevaluates that transport costs are very influential in expenses. If the hotel does not provide transportation for free, then the cost for especially fuel prices will be more expensive.

S3 decision changes concerning hotel ranking is also confirmed from the results of the interview.

*I : During the last discussion Did you find any error?*

**S3:** Yes ma'am

*I: What did you think during the last discussion until it could be found the error?*

**S3:** Yes ma'am I think, eventually the hotel becoming the first choice is the millennium because of the free transport, the close distance, the facility of swimming pool. Because, the playground place is not necessarily a playground that can be used to practice the basketball while the pool can increase stamina of the players. The second option is Santika hotel, and the third option is Pradana hotel. So the total cost of the tournament for the millennium hotel is  $3 \times 1.09 \text{ million} = 3.27 \text{ million}$  because

the match date is 15-16 th of June so they have to stay at the hotel for two nights, so it becomes 3.27 million  $2 = 6.54 \text{ million}$ . Therefore, for the cost per player is  $6.54 \text{ million} \div 10 = 654,000$ , divided by 10 because the experience of the coach also paid.

From the interview above, S3 realizes that the idea of his friends makes sense, that it is necessary to reconsider the relationship between hotel ranking and all the criteria provided in the table as the price per night, maximum persons per room, distance, restaurants, facilities, presence or absence of transportation costs. S3 then evaluates by rethinking about his initial decision in making the hotel ranking, then it makes her do that regulation, that is changing totally his initial procedure in determining the hotel ranking.

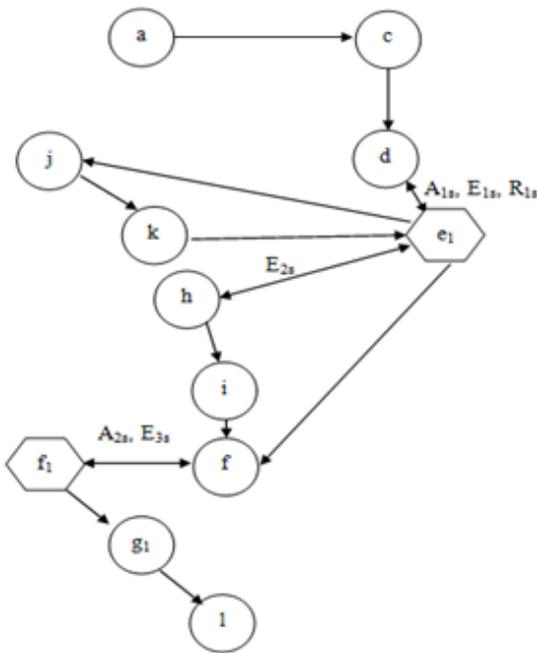
S3 also revises his decision in determining the cost per player for 2 nights. After discussing with her friends, S3 realizes that his friend idea is logical, so it is necessary to reconsider the length of stay. On that problem, it is stated that the match will be held on June 15 to 16. Based on the experience of one member of the group in the tournament, the players arrived a day before the date of the match, so the basketball players normally stayed for two nights. Here is discussion.

**S3:** Wait a minute it was for 2 nights or 2 days ... depending on two nights in two days. It could have been 2 days but in 1 night

**S2:** Hah yes it could be 1 night

**S3:** Yes I think is realistic. It could be the date 14 in that night when we got there, because we go from Malang to Jakarta. The next may not depart continue straight game (Evaluation)

**S1:** But surely we do not know where the school is



**Figure 4.** The thinking structure of S3 after resolving the problem in group.

mentioned

**S3:** that your school team, it means our school team

**S1:** Means is it for 3 times?

**S3:** Yes 2 times

**S1:** Yes means we decide 2 nights

Based on the results of think aloud above, after realizing that his friend idea is logical, S3 performs the evaluation. The evaluation conducted by S3 is rethinking of his initial decision in calculating the cost per player for one night. Based on the experience of one member of the group in the tournament, the players arrived a day before the date of the match, so the basketball players normally stayed for two nights.

Based on the discussions above, S3 experiences constructive metacognitive activities shift from individual to social. This happens due to the influence of group discussion so that students rechange the initial procedure path in determining the hotel ranking. This metacognitive shift involves metacognitive activity components of awareness, evaluation, and regulation. Furthermore, the thinking structure of S3 after resolving the problem in group can be seen in Figure 4.

**Scheme of constructive metacognitive activity shift students in solving problems from individual**

Metacognitive activity appears when S3 solves the problems of decisions either individually or in group

discussion. Ideas or input from discussion partners, provide influence on a person. Influence in group discussions results in the metacognitive activity shift process from individual to social, that is, a condition in which the students re-examine their mathematical thinking and revise their initial solutions in solving the problems (Table 3).

Metacognitive activity shift occurs when S3 tries to rethink about the initial decision in determining the hotel ranking and the cost per player. S3 realizes that feedback and ideas from his friends are logical, therefore, S3 performs the evaluation. The evaluation conducted by S3 is rethinking of his initial decision initially in calculating the cost per player for one night. When solving individual problem, S3 decides that the first option is Pradana hotel because the price is the cheapest among the three hotels, the maximum capacity per room is only a little. It means that player rest can be maximum, there are many restaurants, and there is a pool and a playground that can be used to exercise. The second option is Santika premiere hotel because although it is expensive, the distance from the hotel to match location is close, the third is millennium hotel because the price is quite expensive, the distance from the hotel to match location is far, and there are little facilities. After a discussion and feedback from his friends, S3 realizes the logic of his friend's idea that transport costs are very influential in expenses. If the hotel does not provide transportation for free, then the cost for especially fuel prices will be more expensive. S3 performs evaluation, that is, by rethinking his decision to rank the hotel. S3 ultimately changes his decision that the first hotel chosen is millennium, the second is a premiere hotel, and the third is Pradana. In arranging the ranking of this hotel, S3 changes totally the initial procedure path and eliminate some of the structural components of the initial thinking which makes him wrong in taking decisions.

The shift experienced by S3 belongs to the constructive metacognitive activity shift. Metacognitive activity shift that occurs either because students dismantle again or change the initial procedure path totally that is originally wrong or eliminate some of the structural components of their initial thinking in solving the problem is called constructive metacognitive activity shift. This constructive metacognitive shift involves metacognitive activity components of awareness, evaluation, and regulation.

S3 also revises his decision in determining the cost per player. When solving problems individually, S3 determines the length of stay is one night. S3 procedure to determine the total cost of the tournament is obtained from the registration fee plus the cost of hotel for one night so that  $1,500,000 + 630,000 = 2.13$  million. Furthermore, to determine the cost per player is  $\frac{2,130,000}{10} = 213,000$  (divided by 10 because there are 9 players and 1 coach). Then, after a discussion with his two friends, S3

**Table 3.** Code description of the thinking structure of S3 after resolving the problem in group>

a	: Reading the problem	h	: Considering the presence / absence of transportation costs and distance
c	: Determining 1) order/hotel ranked from best to worst, 2) total cost of the tournament and the hotel, and 3) the cost per player	g <sub>1</sub>	: Calculating the cost per player for 2nights
d	: The cheapest hotel price per night	l	: finished
e <sub>1</sub>	: Comparing the cost of the room/hotel, distance, the number of restaurant, and facilities on each type of hotel	→	: The order of activity when solve the problems
f	: Calculating the total cost of a hotel room and registration	↔	: Activities of alternating or 2 directions
f <sub>1</sub>	: Length of stay for 2 night	A <sub>i</sub>	: Individual awareness
i	: The order of hotel's rank from best to worst	R <sub>i</sub>	: Individual regulation
j	: Determining the need of room for 10 people on of each type of hotel	E <sub>i</sub>	: Individual evaluation
k	: Calculating the room cost for 10 people on each type of hotel		

realizes his friend idea that it is also necessary to reconsider the length of stay. On the problem of a basketball tournament, it is stated that the match will be held on June 15 to 16. Based on experience in the tournament, the players arrive a day before the date of the match, so that members of the group decide that the basketball players normally stay for two nights. S3 performs evaluation, that is, by reconsidering the initial decision in calculating the cost per player for one night. The revised S3 answer after discussion with the group can be seen in Figure 5.

The cost per player for 2 nights is that the total cost of the tournament for the millennium hotel is  $3 \times 1.09$  million = 3.27 million. The match is 15-16<sup>th</sup> June, so the players stay 2 nights, and  $3.27 \text{ million} \times 2 = 6.54$  million. Therefore, the cost for per player is  $6.54 \text{ million} \div 10 = 654,000$ .

Figure 6 is the scheme of constructive metacognitive activity shift of students in solving the problems from individual to social. When the thinking structure (schema) of someone while solving problems individually is not suitable with the structure problem, then during the discussion process (social interaction) it will occur modifying the scheme previous (scheme person when solving problems individually). The modification process involves one's previous scheme to reorganize and complete the previous scheme to form a new scheme so it is more complex and suitable with the structure problem and this process is the integration process (Subanji, 2011). The modification process in social interaction occurs when one is aware again that the previous schemes are not suitable with the problem structure so that person needs reevaluate and regulate again the previous scheme so it is suitable with the problem

structure. The modification process in social interaction occurs when one has been influenced of his friend in discussion group and it led to constructive metacognitive activity shift. This constructive metacognitive activity shift involves three activities metacognitive; they are awareness, regulation, and evaluation.

## DISCUSSION

This study aims to describe the constructive metacognitive activity shift 11th graders in solving a mathematical problem-solving task. To explore the constructive metacognitive activity shift, we used Model Eliciting Activities (MEA). According to Magiera and Zawojewski (2011), MEA is a kind of open problems requiring the development of mathematical models and requires challenges so that members of the group should be involved to decide, test, and revise their initial solution that in turn it leads to the monitoring, evaluation of the effectiveness of their initial solution, and make a decision. Based on the analysis results of MEA problem solving sheet, think aloud outcome, results of recorded conversations of students during the group discussion, questionnaires, and interviews, it is find that the discussions in small groups to solve problems provide influence on other members to examine and revise their initial solutions. Influence in group discussions can lead to constructive metacognitive activity shift. It is the shift due to the influence of a group discussion where students reorganize their initial procedure and complement their answers when solving problems and involves the components of awareness, regulation, and evaluation.

Constructive metacognitive activity shift occurs in S3,



Figure 5. The revised S3's answer after discussion with the group.

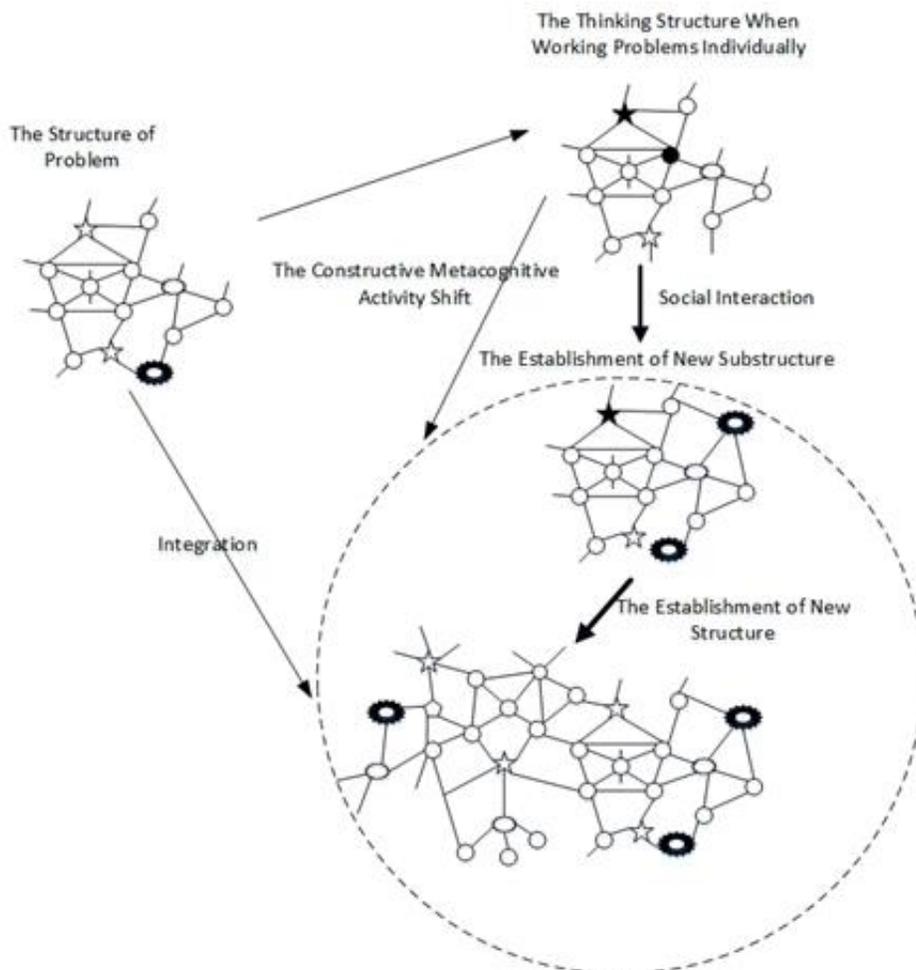


Figure 6. Scheme of constructive metacognitive activity shift .

that is, when he attempts to rethink the initial decision to determine hotel ranking and cost per player as a result of the influence of the discussion.

After a discussion and get feedback from friends, S3 aware about his idea that transport costs are very influential in the expenses so that it will affect the ranking

of the hotel. These findings are in line with one of the social-based characterization stated by Magiera and Zawojewski (2011), interpreting a variety of perspectives. This characterization illustrates how a person's thinking is driven by a mathematical approach of others, examples of considering the new information generated by their

peers and struggling to understand the mathematical explanations presented by others.

During the discussion, S3 also reconsiders relationship between the hotel ranking and criteria provided in the table such as the presence or absence of transportation costs. In rearranging the hotel ranking, S3 experiences constructive metacognitive activity shift in which he changes totally the initial procedures and eliminate some of the structural components of initial thinking which makes him wrong in taking decisions. This constructive metacognitive activity shift involves metacognitive activity components of awareness, evaluation, and regulation. S3 evaluates and rearranges the strategies in determining the hotel ranking. These results are in line with the statement of Magiera and Zawojewski (2011) that indicators of metacognitive regulation include: a) making a plan, b) planning strategy, c) selecting problem-solving strategy, and d) changing the way that has been performed.

## Conclusion

Discussion in small groups makes other members to examine and revise their initial solutions. Discussions of this group results in the process of constructive metacognitive activity shift. It is the shift that occurs due to the influence of group discussion in which students reorganize their initial procedure and complement their answer when solving problems. Based on the data analysis, this constructive metacognitive activity shift is rarely performed by the students and this shift involves three metacognitive activity components of awareness, evaluation, and regulation.

Certainly, the results of the current study mean much and have implications for the development of science. Teachers need to consider discussion groups in problem solving activities. Discussion groups could encourage metacognitive activity that will help them in solving the problem.

## Limitation and further study

The results of this study are limited to the data collected from the eleventh grade students. Therefore, similar studies can be carried out through collecting data from the university students. The results of these studies can be compared to the results of the current study. Also this study has not described the process of perfective metacognitive activity shift. Therefore, further study is needed to describe the process of perfective metacognitive activity shift.

## Conflict of Interests

The author has not declared any conflicts of interest.

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